Optimal multi-carrier link configuration

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Goodput-optimal link configuration

- (Goldsmith, Goodman, et al., 2006 [1]) proposes it for
  - single communication link
  - M-QAM modulation
  - error-detecting codes (CRC)
- performance index: (net) throughput (goodput), given by

$$T = \frac{L - C}{L} b R_s f(b, \gamma_s, L) \tag{1}$$

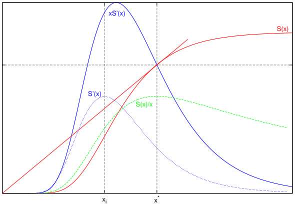
  - L, C : packet length, CRC length in bits
  - b , R_s bits per symbol, symbol rate
  - γ_s : *per symbol* signal-to-noise ratio.
  - $f(b, \gamma_s, L) = [1 - P_b(\gamma_s, b)]^{L/b}$ packet-success rate (1 - PER)
  - $P_b(\gamma_s, b)$ *symbol*-error probability
- Basic idea: choose combination of parameters that (jointly) maximises T

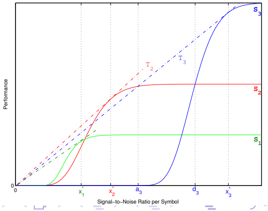
"Goodput"-ideal link configuration

- with packet-success rate $f(x;\mathbf{a})$, & $R = Hp/(N_0 x) \leq \hat{R}$, throughput is:

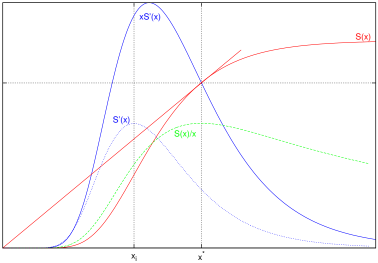
$$\bar{b} R f\left(\frac{Hp}{N_0 R};\mathbf{a}\right) \equiv \frac{Hp}{N_0}\frac{\bar{b} f(x;\mathbf{a})}{x} \qquad (2)$$

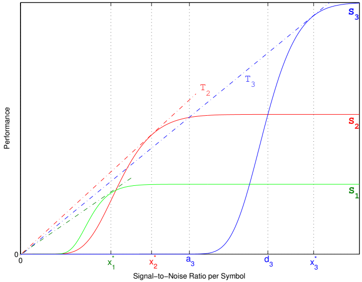
- $\bar{b} f(x;\mathbf{a})/x$ (bps/Hertz) has form $S(x)/x$
- $S(x)/x$ (green "bell" curve) is maximised at the unique tangency point, x^*, between a line from (0,0) and the S-curve ($x^* S'(x^*) = S(x^*)$) [2]
- $\therefore$ configuration with greatest $\rho^* := \bar{b} f(x^*;\mathbf{a})/x^*$ (steepest tangent) maximises bps/Hertz [3]



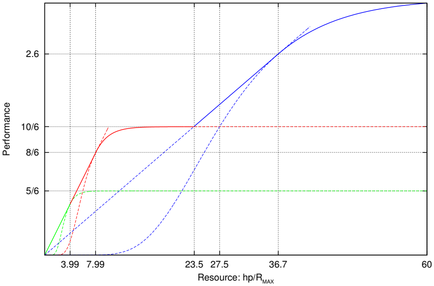


The steeper the tangent the better the configuration



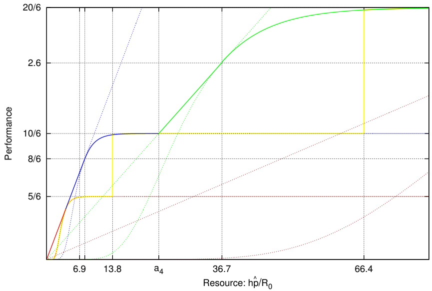


Application to M-QAM ($b \in \{1, 2, 4\}$)



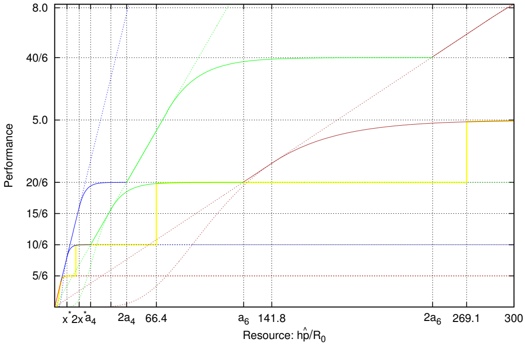
- With $C = 16$ and $L = 96$, BPSK ($b = 1$, green) & QPSK ($b = 2$, red) are tied
- each outperforms 16-QAM ($b = 4$, blue)

Experiment 1: $\hat{R} = R_0$



- $h\hat{p}/R_0 > x^* = 7.99 \implies R^* = h\hat{p}/x^* > R_0 = \hat{R}$; $\therefore R \leftarrow R_0$ & solid blue curve yields performance (see [4])
- For $h\hat{p}/R_0 \geq a_4$, b set to 4, & R set to $R_4^* = h\hat{p}/x_4^*$ to achieve $x_4^* = 36.7$; performance given by solid green line...
- Rate-flex outperforms traditional (yellow steps) by $\approx$ 2-to-1

Experiment 2: $\hat{R} = 2R_0$

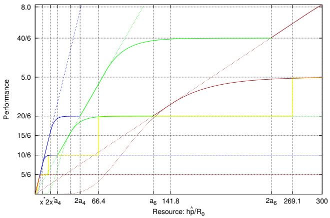


Similar to experiment 1 (lower multicolor), but transitions at $2x^*$, $2a_4$, $2x_4^*$, & $2a_6$. Rate-flex advantage $\geq$ 3-to-1 (see [4]).

Recapitulation

- Previous work recognised the importance of link configuration (modulation, packet size, coding, etc) under higher-layer criteria, but ran into technical obstacles
- Analytical geometry led to a sharp and general result: "the steeper the tangent the better the configuration"
- Here we compared ours vs. "traditional" (modulation-only) link adaptation for M-QAM with binding rate constraint

- large symbol rate constraint $\Longrightarrow$ overwhelming performance edge
- flexible rate $\leq$ fixed rate $\Longrightarrow$ significant edge (up to 2-to-1)
- Intermediate cases follow same pattern



Power allocation to sub-channels ("waterfilling")

- Optimal allocation of power to several subchannels is well understood ("water filling") under 2 conditions:
  - performance $\leftarrow$ Gaussian "capacity" ($\propto \log(1 + \text{SNR})$)
  - power is limited but costless
- In CISS'10 [5] we generalised problem by considering:
  - general capacity function (channel need NOT be Gaussian)
  - co$tly €nergy
- But system rarely at/near capacity, $\therefore$ capacity maximising allocation may NOT maximise "true" performance
- We now assign to sub-channels not only power but ALL LINK PARAMETERS to maximise actual performance (not theoretical capacity) while also considering an energy cost (which could be zero!)

Power and symbol rate optimisation formulation

optimise power, and symbol rate for given configuration

- One terminal, M subchannels, total power constraint $\hat{P}$
- subchannel gains: $H_1 \geq \cdots \geq H_M > 0$
- choose p_m and R_m to maximise benefit minus cost:

$$\max_{\substack{p_1,\cdots,p_m \\ R_1,\cdots,R_m}} b_0 \sum_{m=1}^{M} \bar{b}_m R_m f\left(\frac{H_m p_m}{N_0 R_m}; \mathbf{a_m}\right) - c_0 \sum_{m=1}^{M} p_m \tag{3a}$$

subject to (3b)

$$\sum_{m=1}^{M} p_m \leq \hat{P} \tag{3c}$$

$$p_m \geq 0 \tag{3d}$$

$$0 \leq R_m \leq \hat{R}_m \tag{3e}$$

Problem re-statement

With $c := c_0/b_0$, $\rho^* = \max_{\mathbf{a},x} \bar{b} f(x;\mathbf{a})/x$, $h_m = \rho^* H_m/N_0$, and

$$B(x_m;\mathbf{a_m}) := \frac{1}{\rho^*} \frac{\bar{b}_m f(x_m;\mathbf{a_m})}{x_m} \quad \text{("bpH operating efficiency")}$$

Problem (3) can be re-written as:

$$\max_{\substack{p_1,\cdots,p_m \\ x_1,\cdots,x_m}} \sum_{m=1}^{M} h_m p_m B(x_m;\mathbf{a_m}) - c \sum_{m=1}^{M} p_m \tag{4a}$$

$$\text{subject to: } p_m \geq 0\,,\ x_m \geq 0$$

$$\sum_{m=1}^{M} p_m \leq \hat{P} \tag{4b}$$

$$h_m p_m - \rho^* \hat{R}_m x_m \leq 0 \tag{4c}$$

Fact

$h_m \leq c \implies$ *subchannel m is useless*

Karush-Kuhn-Tucker (KKT) conditions

Fact

If $(x_1, \cdots, x_M)$ and $(p_1, \cdots, p_M)$ solve Problem (4), and $p_m > 0$ then there are non-negative numbers $\lambda_0, \mu_1, \cdots .\mu_M$ such that

$$h_m (B(x_m; \mathbf{a_m}) - \mu_m) = c + \lambda_0 \quad (5a)$$

$$h_m p_m B'(x_m; \mathbf{a_m}) + \rho^* \hat{R}_m \mu_m = 0 \quad (5b)$$

$$\lambda_0 \left(P - \sum p_j\right) = 0 \quad (5c)$$

$$\mu_m \left(h_m p_m - \rho^* \hat{R}_m x_m\right) = 0 \quad (5d)$$

KKT "issues"

- Should a given channel be used ($p_m > 0$)?
- "Complementary slackness" issues:
  - If yes, should it operate at the maximal symbol rate ($\mu_m > 0 \implies R_m = \hat{R}_m$)?
  - Should all available power be used ($\sum p_j < \hat{P} \implies \lambda_0 = 0$)?

Two key KKT Facts

- The following fact follows directly from KKT (5a-5b):

Fact

At most one channel operates with $\mu_m = 0$ ($R_m < \hat{R}_m$).

- The proof of the following fact is in the paper

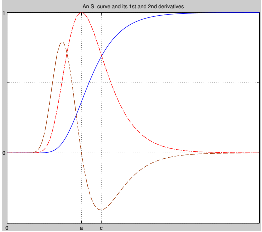
Fact

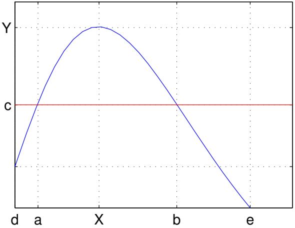
If $p_m > 0$ and additionally $\mu_m > 0$ (that is, $R_m = \hat{R}_m$), then

$$\frac{H_m}{N_0}\bar{b}_m f'(x_m; \mathbf{a_m}) = c + \lambda_0 \tag{6}$$

Solution for maxed-out channel

- (6) $((H_m/N_0)\bar{b}_m f'(x_m;\mathbf{a_m}) = c + \lambda_0)$ is critical b/c at most 1 active channel has $R_m < \hat{R}_m$
- f is an S-curve, $\therefore$ its derivative (dash-dot $\rightarrow$) is "single peaked", & so is the left of (6)
- Hence, (6) has at most 2 solutions (see a & b $\rightarrow$)
- if left solution was chosen, greater $H_m \Rightarrow$ lower x_m





KKT summary

- if a (normalised) channel gain $<$ normalised power cost "throw away" channel
- If power is "scarce" give all to best channel, and use ideal (single-channel) configuration
- for at most one n, $R_n < \hat{R}_n$, & n uses the ideal configuration
- if there is "left over" power (which is possible), then all active channels operate at maximal symbol rate
- The main result is Fact 4 (SNR of max-rate channel is the larger of the at most 2 solutions of a simple equation (6)):

$$\frac{H_m}{N_0}\bar{b}_m f'(x_m; \mathbf{a_m}) = c + \lambda_0$$

- Major KKT outstanding issue: find λ_0 in (6)

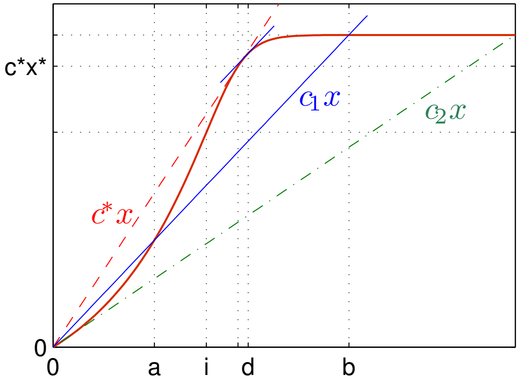
Economic interpretation of FONOC

- Suppose that for each channel is represented by a “selfish agent” which can buy power at a unit cost $c + \lambda_0$
- with $x = H_m p/(N_0 \hat{R})$ agent maximises “benefit minus cost”

$$\bar{b}_m \hat{R} f(x; \mathbf{a_m}) - (c + \lambda_0) x \hat{R}/(H_m/N_0) \qquad (7)$$

- which leads to $\bar{b}_m f'(x; \mathbf{a_m}) = (c + \lambda_0)/(H_m/N_0) \equiv$ (6)
- m obtains the largest solution to (6), x_m, from which it gets a power level: $p_m = x_m \hat{R}/(H_m/N_0)$
- But for arbitrary λ_0, their total “demand” may exceed “supply” (power constraint), or leave unused power
- Below we search for the right λ_0, and also identify the best configuration per channel

Maximising S(x)-cx s.t. x≤X

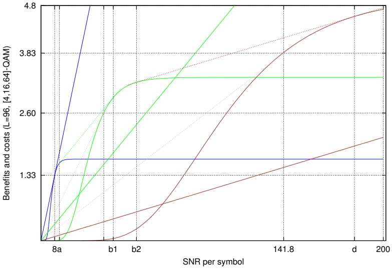


If $c > c^*$ or $c = c_1$ and $X < a$ then $x = 0$ is optimal. Otherwise $\min(X, x^*)$ is the maximiser. At x^*, the curve's tangent (short blue line) is parallel to the cost line cx.

Maximising S(x)-cx & choosing S

- When cx lies on green ray, then a and b_1 are optimal choices for curves S_1 and S_2, that is, $S_1'(a) = S_2'(b_1) = c$
- Also, $S_1(a) - ca = S_2(b_1) - cb_1$ (same “utility” with either!)

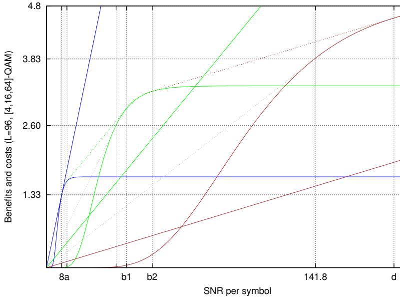
- cost line left of green $\Longrightarrow$ S_1 is better
- cost line right of green $\Longrightarrow$ S_2 is better
- $\Longrightarrow$ configuration criterion!



Systematic search for the Lagrange multiplier

- Fo $c+\lambda_0$ sufficiently small, agent gets x where $\bar{b}_m f'(x;\mathbf{a_m}) = c_m = (c+\lambda_0)/(H_m/N_0))$
- To find λ_0^* "sweep" price line from vertical to horizon
- $H_m > H_n \implies c_m < c_n \therefore$ best channel is first to "buy"

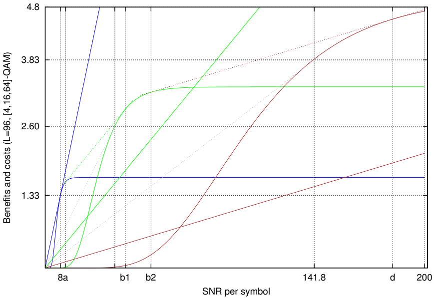
- If $c_1 = (c+\lambda_0)/(H_1/N_0) >$ slope of blue line, all choose 0
- If c_1= slope of blue line only channel 1 buys (x at blue "knee")
- If $c_2 = (c+\lambda_0)/(H_2/N_0) \leq$ slope of blue line channel 2 also buys



Configuration choice

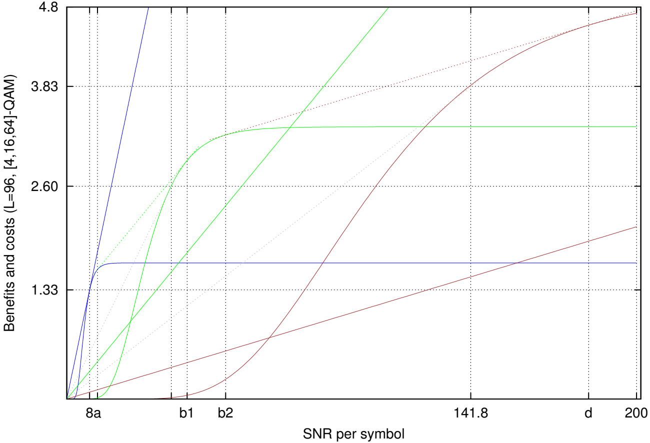
- Green line marks transition: If $c_m x$ coincides with green line, then m can get same performance with blue or green curve (benefit minus cost is the same for either curve).

- m goes green when c_m falls just under slope of green line
- Brown line marks similar green-to-brown switch



- process ends when the SNR values chosen by the agents lead to power levels that exactly add up to available power

Green line marks transition



If cx is green line, $S_1'(a) = S_2'(b_1) = c$ & $S_1(a) - ca = S_2(b_1) - cb_1$

Experimental set-up

- For {4,16,64}-QAM, with $L = 96$ and $C = 16$
  - slopes of the tangenus:
    $\rho_1^* = 0.166$, $\rho_2^* = 0.071$ and $\rho_3^* = 0.027$
  - transition prices (slopes of green and brown lines)
    $c_{12} = 0.042$ and $c_{23} = 0.011$
- Let the experimental parameter $\Pi \in [10, 200]$ be:

$$\frac{H_1}{N_0} \frac{\hat{P}}{\hat{R}} := \Pi$$

- Let $H_m := h(\alpha, m) H_1$ with $h(\alpha, m) := \alpha^{m-1}$ with $0 < \alpha < 1$ and $m \in \{1, \ldots, M\}$ with $M = 5$
- Let $\pi := (\pi_1, \ldots, \pi_M)$ with $\pi_m \in [0, 1]$ and $\sum \pi_m = 1$ denote some heuristic allocation rule. We consider:
  - egalitarian allocation: $\pi_m = 1/M$ and
  - quality-proportional allocation: $\pi_m = H_m / \sum_{j=1}^{M} H_j$

Experimental results (inner details)

Table: KKT allocation details

$\alpha = \frac{31}{32}$			$\alpha = \frac{1}{2}$			$\alpha = \frac{1}{4}$		
$h_{\alpha,m}$	$\frac{x_m}{h_{\alpha,m}}$	b	$h_{\alpha,m}$	$\frac{x_m}{h_{\alpha,m}}$	b	$h_{\alpha,m}$	$\frac{x_m}{h_{\alpha,m}}$	b
1	43.8	4	1	47.7	4	1	183.4	6
$\frac{31}{32}$	44.9	4	$\frac{1}{2}$	68.7	4	$\frac{1}{4}$	16.6	<2
0.94	46.1	4	$\frac{1}{4}$	25.6	2	$\frac{1}{16}$	0	—
0.91	47.2	4	$\frac{1}{8}$	37.1	2	$\frac{1}{64}$	0	—
0.88	17.9	<2	$\frac{1}{16}$	20.1	<2	$\frac{1}{256}$	0	—

$< 2 \implies$ not enough resource to operate a max rate

Experimental results: performance

Table: KKT performance vs. 2 heuristics

α	$\bar{c}^*$	KKT-Perf	Q-perf	Eg-Perf	%
$\frac{31}{32}$	0.0358	13.53	8.09	8.22	65
$\frac{5}{8}$	0.0246	9.60	6.58	5.67	69
$\frac{1}{2}$	0.0197	8.15	6.03	4.46	83
$\frac{3}{8}$	0.0149	6.92	5.13	3.56	94
$\frac{1}{4}$	0.0101	5.27	4.82	2.33	126

Summary

- Found throughput-maximising link configuration (power, modulation family, order, packet length, etc.) for several sub-channels with possible power cost.
- Simple solution algorithm possibly viewed and implemented as a “pricing game” played by software agents, each representing a sub-channel
- Numerical experiments yield performance gains over two simple heuristics of up to 126 percent.
- CONCLUSION: solved a highly dimensional, complex and important problem in a relatively simple manner, and the reported numerical results are highly encouraging.

For Further Reading I

- T. Yoo, R. J. Lavery, A. Goldsmith, and D. J. Goodman, "Throughput optimization using adaptive techniques." http://wsl.stanford.edu/, 2006.
- V. Rodriguez, "An analytical foundation for resource management in wireless communication," in *Global Telecommunications Conf.(GLOBECOM), IEEE*, vol. 2, pp. 898–902 Vol.2, Dec. 2003.
- V. Rodriguez and R. Mathar, "Generalised link-layer adaptation with higher-layer criteria for energy-constrained and energy-sufficient data terminals," in *Wireless Communication Systems (ISWCS), 7th Inter. Symp. on*, pp. 927 –931, 2010.

For Further Reading II

- V. Rodriguez, “Generalised link-layer optimisation: Application and performance evaluation,” in *Cross Layer Design, Third Inter. Workshop on*, pp. 1 –6, dec. 2011.
- V. Rodriguez and R. Mathar, “Generalised water-filling: costly power optimally allocated to sub-carriers under a general concave performance function,” in *Information Sciences and Systems (CISS), 44th Annual Conference on*, pp. 1 –3, 2010.