LTE Downlink Radio Scheduler - Analytical Modeling

FFV, Aachen - March 16th, 2012

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Problem Statement

- This work targets the analytical modeling of the LTE MAC scheduler
- The presented model is an extension of the model in [1]
- The scheduler has a fixed amount of Physical Resource Blocks (PRBs)
- The scheduler distributes the PRBs among the active users every TTI [1]
- The UEs have different SINR i.e. can support different Modulation and Coding Scheme (MCS)

[1] S. Doirieux, ... "An efficient analytical model for the dimensioning of WiMAX networks supporting multi-profile best effort traffic", Computer Communications, Volume 33, Issue 10, 15 June 2010

[1] Transmission Time Interval = 1ms

LTE Time Domain Schedulers

MaxT: Maximum Throughput

$$P_k(t)_{MaxT} = argmax_k \left[SINR_k[t]\right]$$

where $P_k(t)_{MaxT}$ is the time domain priority factor, k is the user number and $SINR_k[t]$ is the instantaneous SINR of user k.

BET: Blind Equal Throughput

$$P_k(t)_{BET} = argmax_k \left[\frac{1}{\overline{\theta_k[t]}}\right]$$

where $\overline{\theta_k[t]}$ is the normalized average throughput of user k.

LTE Frequency Domain Scheduler

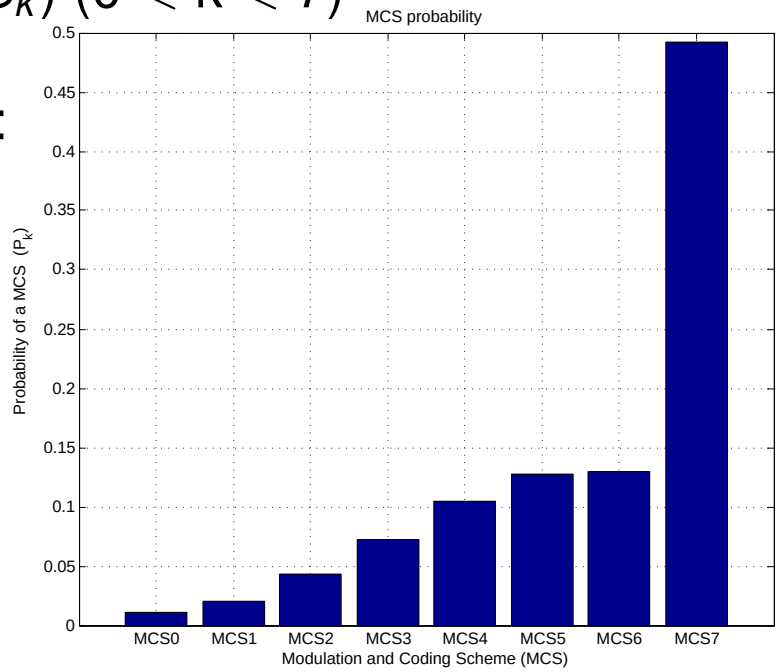
- The FDS is round robin scheduler
- It serves up to ψ=5 users per TTI
- This means, that the UEs served per TTI are equal to η:

$$\eta = min(n - n_0, \psi]$$

with n being the number of active users in a TTI, and n_0 is the number of users in outage

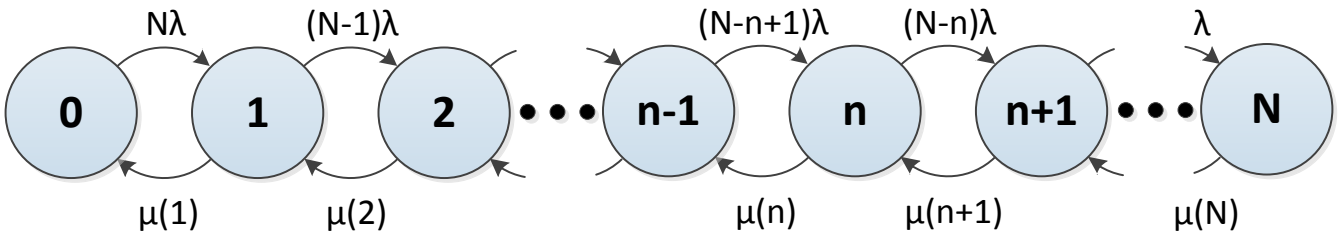
Model Assumptions

- A UE can use the whole spectrum
- UEs are configured with only one bearer
- UEs are statistically identical with respect to the channel
- The channel is divided into 8 states (MCS_k) ($0 < k < 7$)
- Each MCS_k has a static probability P_k
- UEs use the same ON/OFF elastic traffic:
  - $\overline{X_{ON}}$ is the average file size in bits
  - $\overline{t_{off}}$ is the average OFF duration in s



General Analytical Model

- The model is based on the Time Contiguous Markov Chain with a state n representing the number of active UEs in a TTI
- The resulting CTMC is made of N+1 states
  - Arrival transition is performed with rate $(N-n)\lambda$ with $\lambda = 1/\overline{t_{off}}$
  - Departure transition is performed with a generic rate $\mu(n)$



Steady State Probability

- Provided that $\mu(n)$ can be estimated, the steady state probability $\pi(n)$ can be calculated from:

$$\boldsymbol{\pi} \cdot \mathbf{Q} = 0 \tag{1}$$

  where $\boldsymbol{\pi}$ is a vector representing the steady state probability, i.e, $\boldsymbol{\pi}$=$[\pi_0, \pi_1, ...\pi_N]$ and $\sum_i \pi_i$ = 1
- **Q** is the infinitesimal generator matrix.

Performance Parameters

- The average number of active users $\overline{Q}$ is expressed as:

$$\overline{Q} = \sum_{N}^{n=1} n\pi(n)$$

- The mean number of departures by unit of time is $\overline{D}$ and is obtained as:

$$\overline{D} = \sum_{N}^{n=1} \mu(n)\pi(n)$$

- According to Little's law, the average ON period duration $\overline{t_{ON}}$ (i.e. duration of an active transfer):

$$\overline{t_{on}} = \frac{\overline{Q}}{\overline{D}}$$

Departure Rate $\mu(n)$

$$\mu(n) = \frac{\overline{TBS}(n)}{\overline{X_{on}} \cdot TTI} \tag{2}$$

- $\overline{TBS}(n)$ is the average amount of bits sent by all served users within a TTI
- $\overline{X_{on}}$ is the average file size of the ON period
- TTI is the Transmission Time Interval (1 ms in LTE)

Generic Average Bit Rate $\overline{TBS}(n)$

$$\overline{TBS}(n) = \sum_{\substack{(n_0,\ldots,n_K)=(0,\ldots,0)| \\ n_0+\ldots+n_K=n \\ n_0\neq n}}^{(n,\ldots,n)} \overline{TBS}(n_0,\ldots,n_K) \binom{n}{n_0,\ldots,n_K} \left[\prod_{k=0}^{K} P_k^{n_k}\right] \tag{3}$$

- $\binom{n}{n_0,\ldots,n_K}$ is the multinomial coefficient
- P_k is the stationary probability of MCS_k
- n_k is the number of active users in MCS_k
- $\overline{TBS}(n_0,\ldots,n_K)$ is the total bits transmitted for all served users under a certain combination $(n_0,\ldots,n_K)$

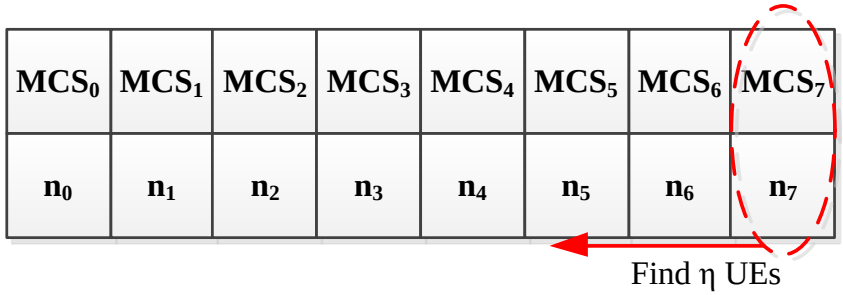
$\overline{TBS}(n_0, ..., n_K)$

$$\overline{TBS}(n_0, ..., n_K) = \sum_{i=[i_1,...,i_\eta]} TBS_i(\eta) \tag{4}$$

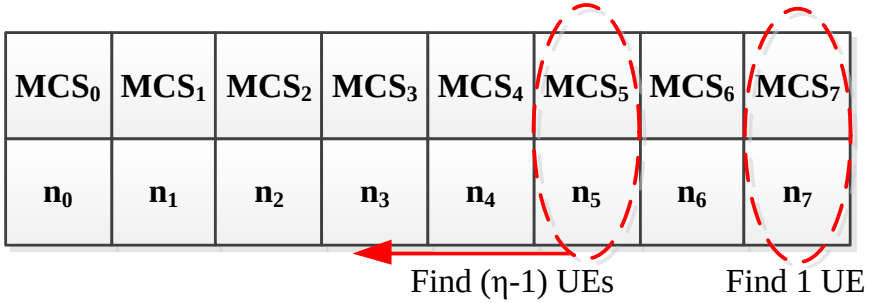
- η is the number of users served under $(n_0, ..., n_K)$
- $[i_1, ..., i_\eta]$ represents index of the chosen η users out of the n active users considered for scheduling within a TTI
- $TBS_i(\eta)$ is transmitted bits of user i using his *MCS* (when η users are served)
- the choice of the η users is determined by the TD scheduler.

Chosen η UEs Procedure

- For the MaxT scheduler the η chosen users are the ones with the highest MCS, and represent the i users indices ($[i_1, ..., i_\eta]$).



- For the BET scheduler the η users are chosen as follows:



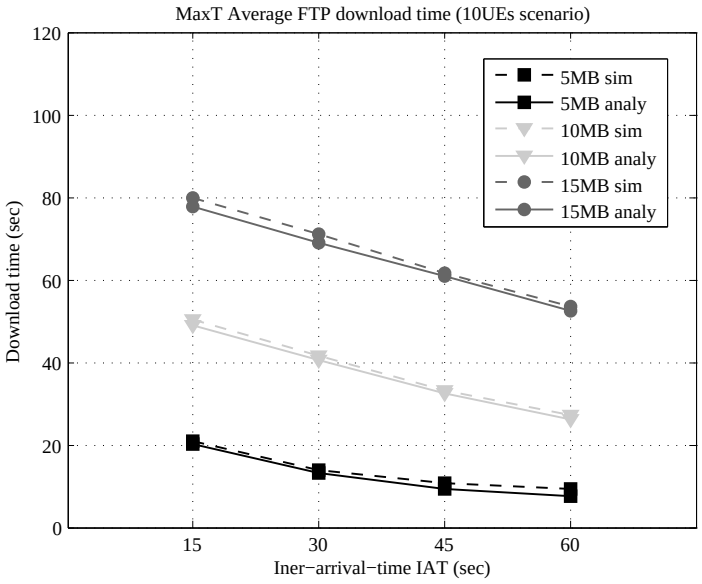
Scenarios Configuration

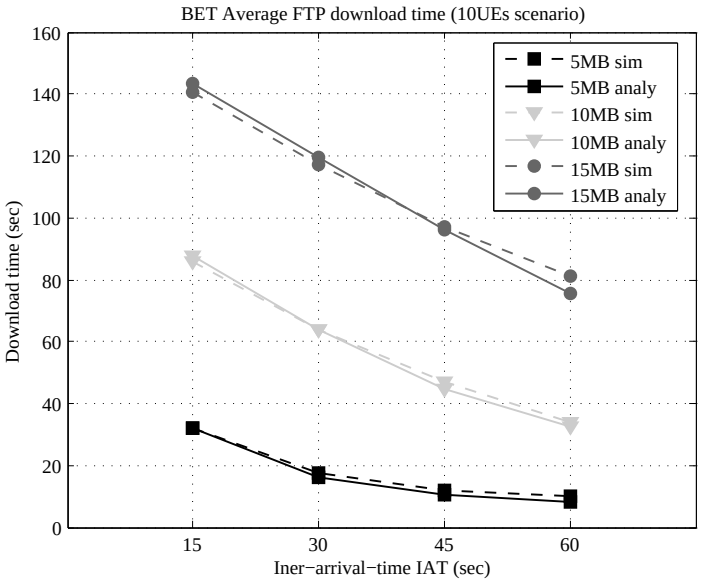
- The results of the analytical model is compared against the OPNET simulation model

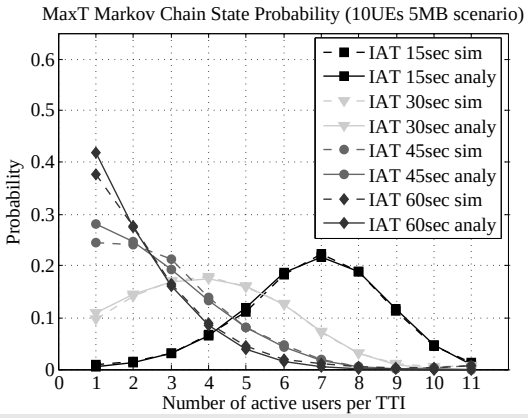
General parameter	Value
Bandwidth (# PRBs)	5 MHz (i.e., 25 PRBs)
Traffic model	FTP with different IATs and file sizes
Simulation run time	8000 s, 10 seeds with 95% confidence interval

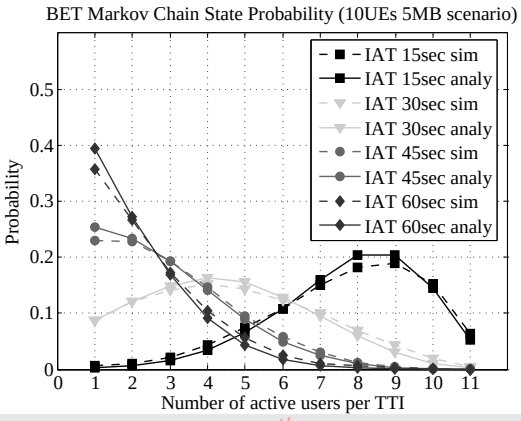
Analysis	Scheduler	# of UEs	FTP file size	FTP IAT
Analysis1	MaxT	10 UEs	constant 5,10 and 15 MB	uniform (0, 30) s, uniform (15, 45) s, uniform (30, 60) s & uniform (45, 75) s
Analysis2	BET	10 UEs	constant 5,10 and 15 MB	uniform (0, 30) s, uniform (15, 45) s, uniform (30, 60) s & uniform (45, 75) s

Average FTP Download Time





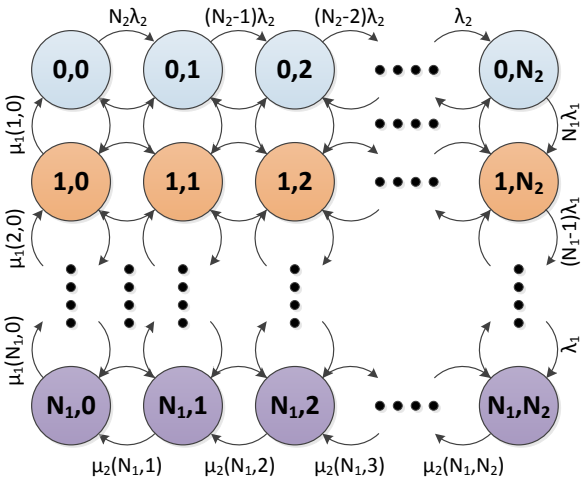


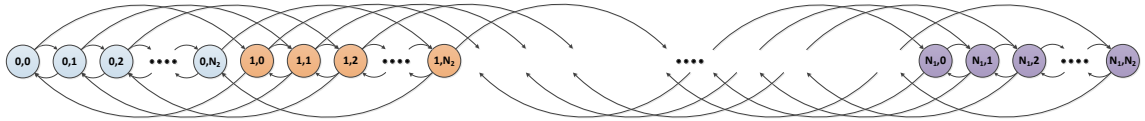


Two Classes Model

- The single class model is extended into a two dimensional Markov Chain, with state (i,j)
  - i represents the number of UEs within class 1
  - j represents the number of UEs within class 2
- The combinations within each state will increase to:

$$(i_0, i_1, i_2, i_3, i_4, i_5, i_6, i_7, j_0, j_1, j_2, j_3, j_4, j_5, j_6, j_7)$$
$$\text{where } \sum_r i_r = N1 \text{ and } = \sum_r j_r = N2$$





Departure Rate $\mu 1(n)$

$$\mu 1(i,j) = \frac{\overline{TBS1}(i,j)}{\overline{X1_{on}} \cdot TTI}$$

$$\overline{TBS1}(i,j) = \sum_{\substack{(i_0,\ldots,i_K,j_0,\ldots,j_K)=(0,\ldots,0)| \\ i_0+\ldots+i_K=i \\ j_0+\ldots+j_K=j \\ i_0 \neq i \text{ with } j_0 \neq j}}^{(i,\ldots,i,j,\ldots,j)} i!j!\overline{TBS1}(i_0,\ldots,i_K,j_0,\ldots,j_K)\left[\prod_{k=0}^{K} \frac{P_k^{i_k} P_k^{j_k}}{i_k! j_k!}\right]$$

$$\overline{TBS1}(i_0,\ldots,i_K,j_0,\ldots,j_K) = \sum_{r=[r_1,\ldots,r_{\delta 1}]} TBS_r(\eta)$$

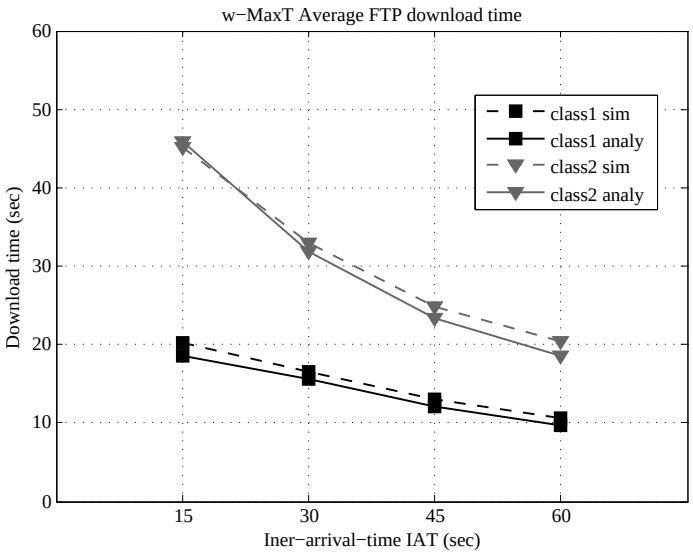
with $r_{\delta 1} + r_{\delta 2} = \eta$

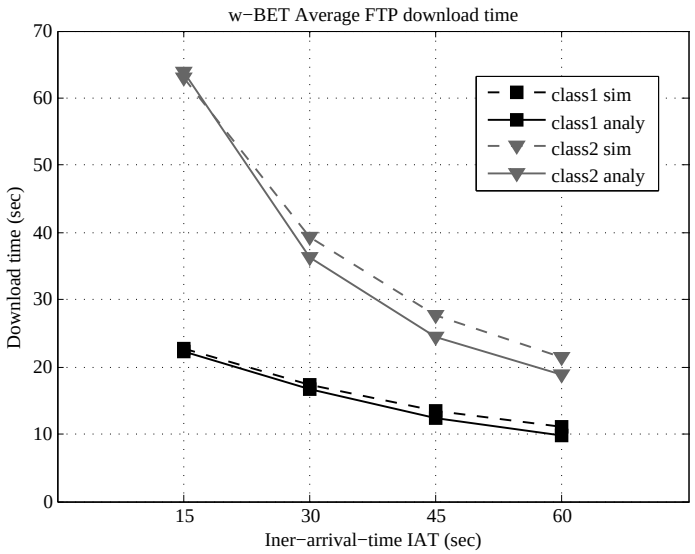
Scenarios Configuration

General parameter	Value
Bandwidth (# PRBs)	5 MHz (i.e., 25 PRBs)
Radio QoS weights	W_{QoS1}=2 and W_{QoS2}=1
Number of users	9 UEs (3 in class1 & 6 in class2)
Traffic model	FTP with different IATs and file sizes
Simulation run time	8000 s, 10 seeds with 95% confidence interval

Analysis	C1 FTP file size	C2 FTP file size	FTP IAT
Analysis1 w-MaxT	constant 5MByte	Constant 10 MByte	uniform (0, 30) s, uniform (15, 45) s, uniform (30, 60) s & uniform (45, 75) s
Analysis2 w-BET	constant 5MByte	Constant 10 MByte	uniform (0, 30) s, uniform (15, 45) s, uniform (30, 60) s & uniform (45, 75) s

Average FTP Download Time





Conclusion & Outlook

- Two different TDS (MaxT and BET) are modeled analytically for single class case
- The analytical model results match the simulation ones
- The w-MaxT and w-BET TDS are modeled with two classes and QoS differentiation
- Results of the 2D model match the simulation results
- The two classes model can be extended for other TDS (e.g., w-PF)
- Additional GBR class can also be modeled

References

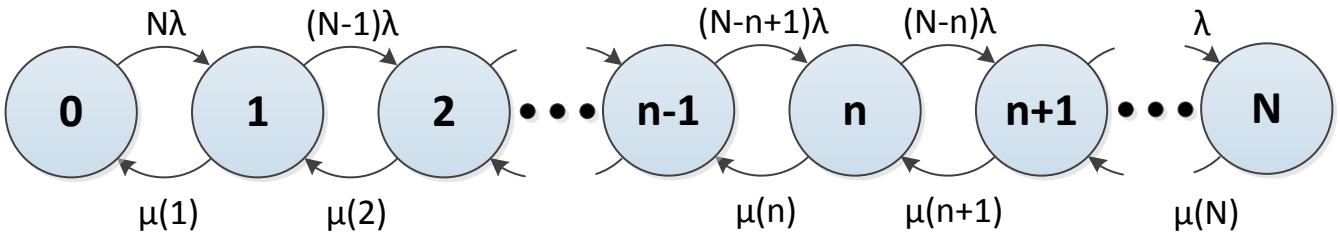
1. S. Doirieux, B. Baynat, M. Maqbool, M. Coupechoux, "**An efficient analytical model for the dimensioning of WiMAX networks supporting multi-profile best effort traffic**", Computer Communications, Volume 33, Issue 10, 15 June 2010, Pages 1162-1179.

Thanks for Listening
Any Question ??

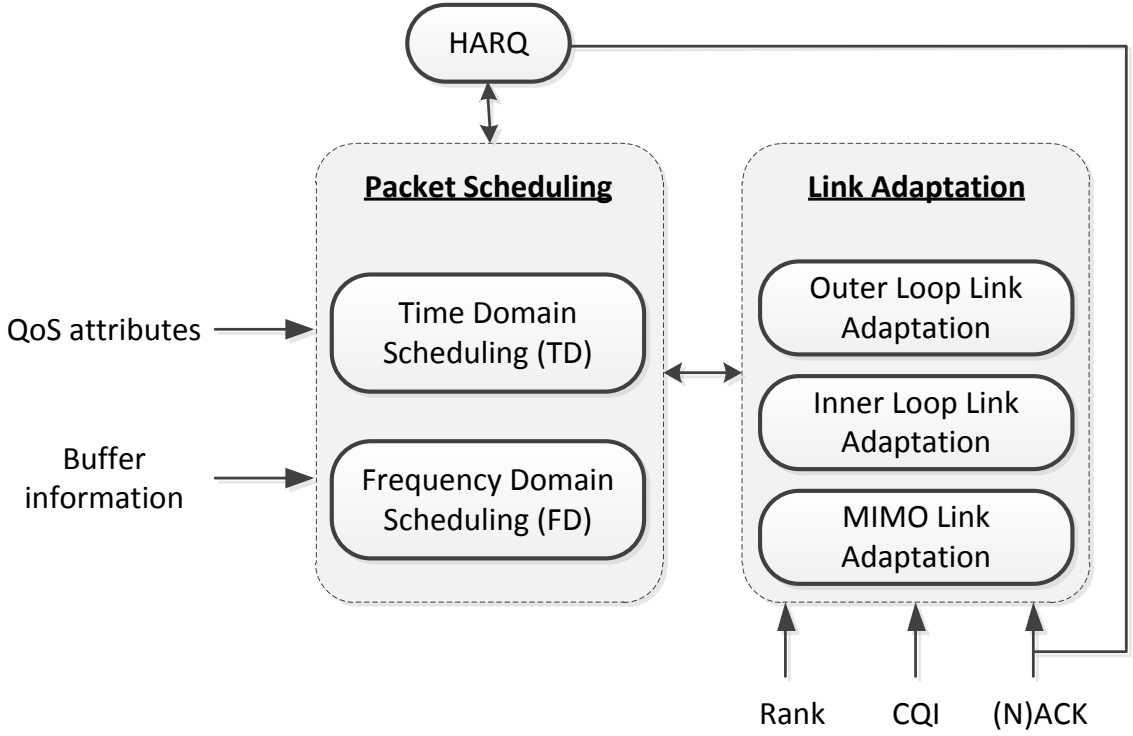
BACKUP SLIDES

General Analytical Model

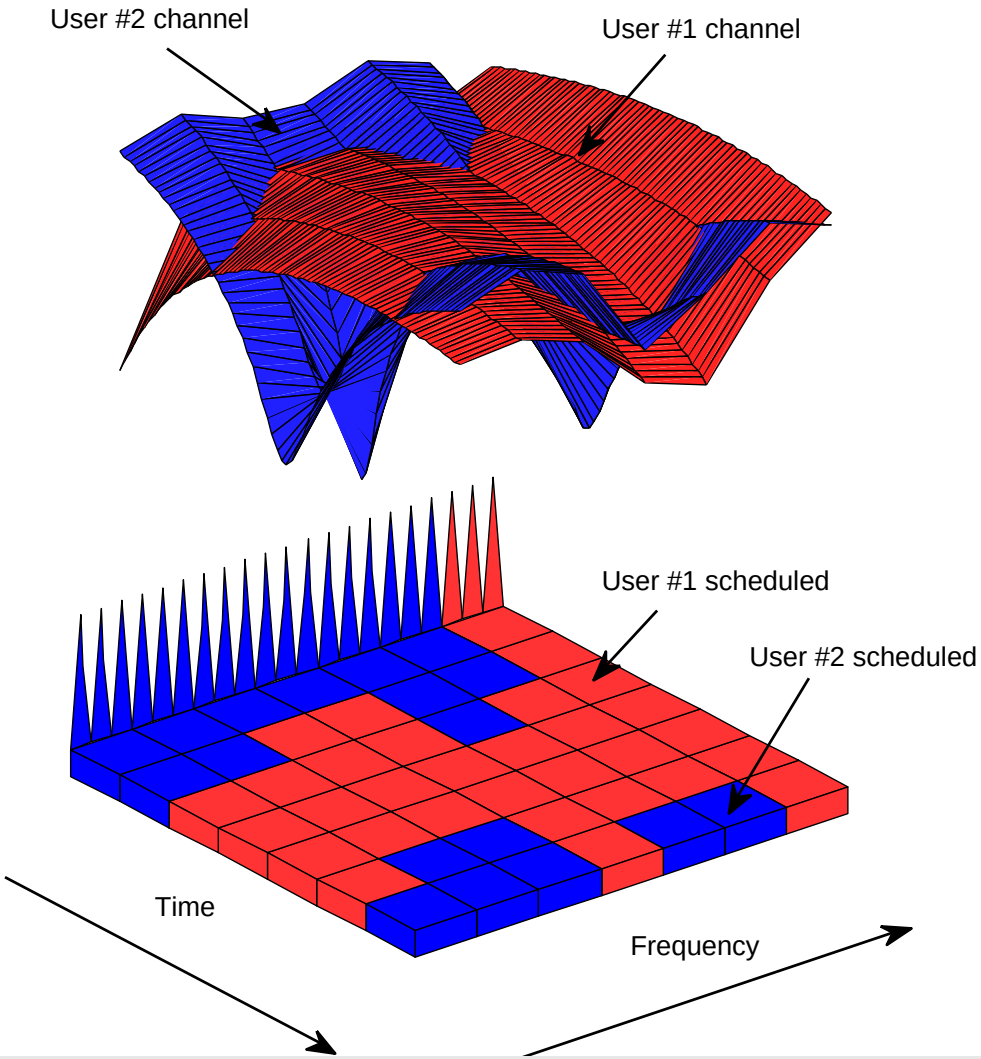
- A transition from state n –> n+1 occurs when a UE in OFF period ends its reading time and initiate a data transfer
  - Arrival transition is performed with rate $(N-n)\lambda$ with $\lambda = 1/\overline{t_{off}}$
- A transition from state n –> n-1 occurs when a UE in ON period complete its transfer
  - Departure transition is performed with a generic rate $\mu(n)$
  - The difficulty lies in estimating the departure rate $\mu(n)$



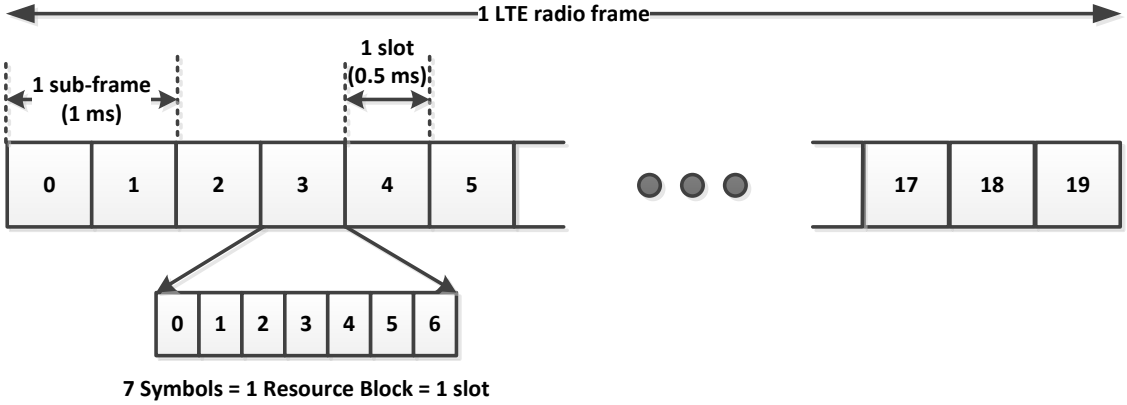
Packet Scheduling Framework

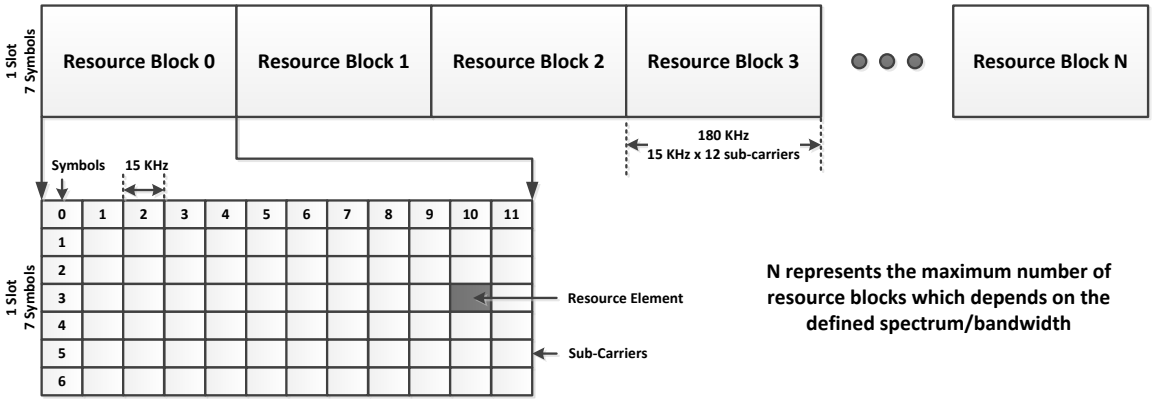


Channel Dependent Scheduling



LTE Radio Frame





Steady State Probability

- Another solution wiil be to use the closed loop solution from the birth-and-death process of the Markov chain:

$$\pi(n) = \left[\prod_{i+1}^{n} \frac{(N - i + 1)\lambda}{\mu(i)} \right] \pi(0)$$

- $\pi(0)$ is obtained by normalization

$\overline{TBS}(2)$

- if we have two active UEs i.e. n=2
- those two users can be at any MCS (from MCS_0 to MCS_7)
- if we represent the number of users in each MCS_k by n_k we get: $(n_0, n_1, n_2, n_3, n_4, n_5, n_6, n_7)$ with $n_0 + ... + n_7 = n$
- then we have the following possible combinations

	MCS0	MCS1	MCS2	MCS3	MCS4	MCS5	MCS6	MCS7
Comb1	0	0	0	0	0	0	0	2
Comb2	0	0	0	0	0	0	1	1
Comb3	0	0	0	0	0	0	2	0
Comb4	0	0	0	0	0	1	0	1
Comb5	0	0	0	0	0	1	1	0
Comb6	0	0	0	0	0	2	0	0
Comb7	0	0	0	0	1	0	0	1
Comb8	0	0	0	0	1	0	1	0
Comb9	0	0	0	0	1	1	0	0
Comb10	0	0	0	0	2	0	0	0
Comb11	0	0	0	1	0	0	0	1
Comb12	0	0	0	1	0	0	1	0
Comb13	0	0	0	1	0	1	0	0
Comb14	0	0	0	1	1	0	0	0
Comb15	0	0	0	2	0	0	0	0

$$\overline{TBS}(2) = \sum_{\substack{(n_0,\ldots,n_K)=(0,\ldots,0)| \\ n_0+\ldots+n_K=0 \\ n_0\neq 2}}^{(2,\ldots,2)} \overline{TBS}(n_0,\ldots,n_K) \binom{2}{n_0,\ldots,n_K} \left[\prod_{k=0}^{K} P_k^{n_k}\right] \tag{5}$$

	MCS0	MCS1	MCS2	MCS3	MCS4	MCS5	MCS6	MCS7
Comb1	0	0	0	0	0	0	0	2
Comb2	0	0	0	0	0	0	1	1
Comb3	0	0	0	0	0	0	2	0
Comb4	0	0	0	0	0	1	0	1
Comb5	0	0	0	0	0	1	1	0
Comb6	0	0	0	0	0	2	0	0
Comb7	0	0	0	0	1	0	0	1
Comb8	0	0	0	0	1	0	1	0
Comb9	0	0	0	0	1	1	0	0
Comb10	0	0	0	0	2	0	0	0
Comb11	0	0	0	1	0	0	0	1
Comb12	0	0	0	1	0	0	1	0
Comb13	0	0	0	1	0	1	0	0
Comb14	0	0	0	1	1	0	0	0
Comb15	0	0	0	2	0	0	0	0
Comb16	0	0	1	0	0	0	0	1
Comb17	0	0	1	0	0	0	1	0
Comb18	0	0	1	0	0	1	0	0
Comb19	0	0	1	0	1	0	0	0
Comb20	0	0	1	1	0	0	0	0
Comb21	0	0	2	0	0	0	0	0
Comb22	0	1	0	0	0	0	0	1
Comb23	0	1	0	0	0	0	1	0
Comb24	0	1	0	0	0	1	0	0
Comb25	0	1	0	0	1	0	0	0
Comb26	0	1	0	1	0	0	0	0
Comb27	0	1	1	0	0	0	0	0
Comb28	0	2	0	0	0	0	0	0
Comb29	1	0	0	0	0	0	0	1
Comb30	1	0	0	0	0	0	1	0
Comb31	1	0	0	0	0	1	0	0
Comb32	1	0	0	0	1	0	0	0
Comb33	1	0	0	1	0	0	0	0
Comb34	1	0	1	0	0	0	0	0
Comb35	1	1	0	0	0	0	0	0

Average number of bits transmitted per PRB under a certain combination

Prob. of a combination.

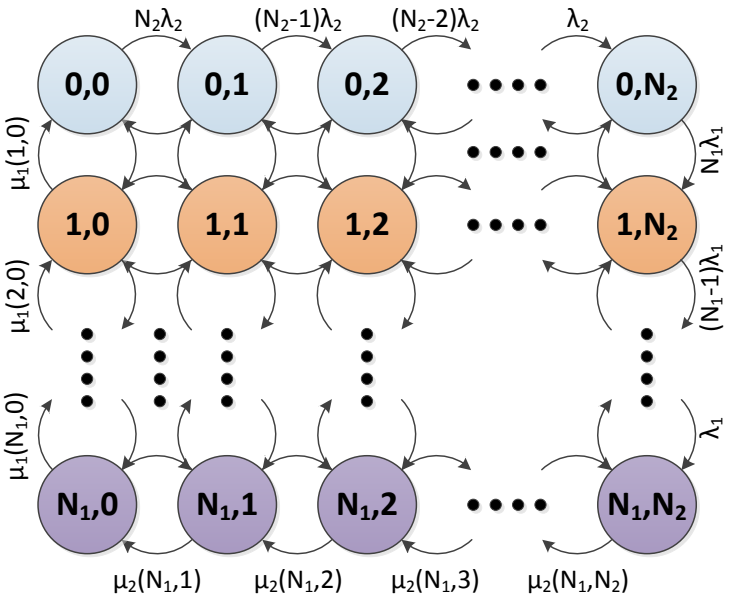
Sum of all probabilities of each combination is equal to 1.

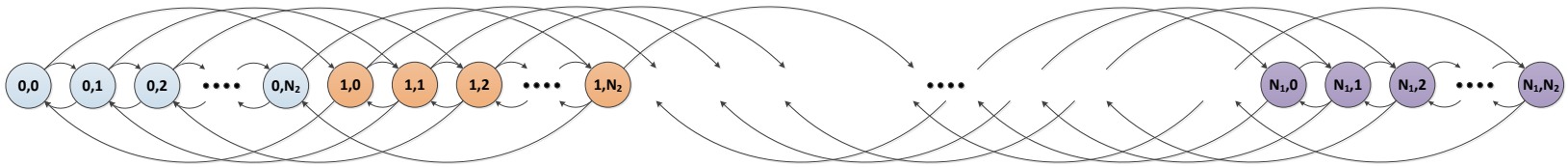
Single Class Model

$$\mu(n) = \frac{1}{\overline{X_{on}} \cdot TTI} \sum_{\substack{(n_0,\ldots,n_K)=(0,\ldots,0)| \\ n_0+\ldots+n_K=0 \\ n_0 \neq n}}^{(n,\ldots,n)} n! \left[\sum_{i=[i_1,\ldots,i_\eta]} TBS_i(\eta) \right] \left(\prod_{k=0}^{K} \frac{P_k^{n_k}}{n_k!} \right) \quad (6)$$

- In order to model the different TD schedulers, a procedure is required to choose the η users.

Q Infinitesimal Generator Matrix



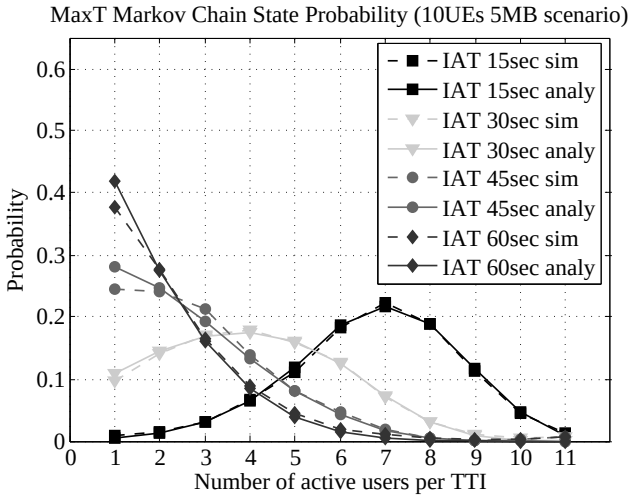


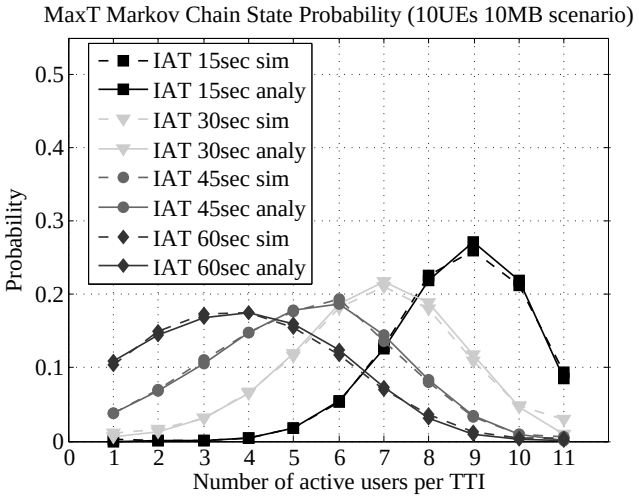
Q Infinitesimal Generator Matrix

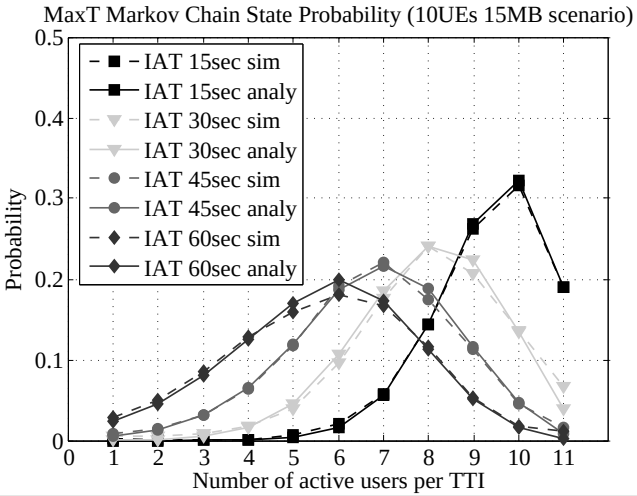
	0 0	0 1	0 2	0 3	0 4	1 0	1 1	1 2	1 3	1 4	2 0	2 1	2 2	2 3	2 4
0 0		4λ1				2λ2									
0 1	μ1 (01)		3λ1				2λ2								
0 2		μ1 (02)		2λ1				2λ2							
0 3			μ1 (03)		λ1				2λ2						
0 4				μ1 (04)		0				2λ2					
1 0	μ2 (10)				μ1 (10)		4λ1				λ2				
1 1		μ2 (11)				μ1 (11)		3λ1				λ2			
1 2			μ2 (12)				μ1 (12)		2λ1				λ2		
1 3				μ2 (13)				μ1 (13)		λ1				λ2	
1 4					μ2 (14)				μ1 (14)		0				λ2
2 0						μ2 (20)				μ1 (20)		4λ1			
2 1							μ2 (21)				μ1 (21)		3λ1		
2 2								μ2 (22)				μ1 (22)		2λ1	
2 3									μ2 (23)				μ1 (23)		λ1
2 4										μ2 (24)				μ1 (24)	

$$Q(i,i) = -\sum_{j} Q(i,j)$$

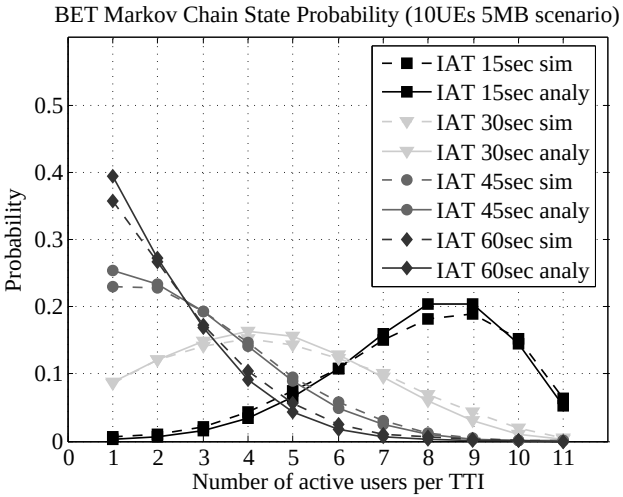
MaxT: State Probability

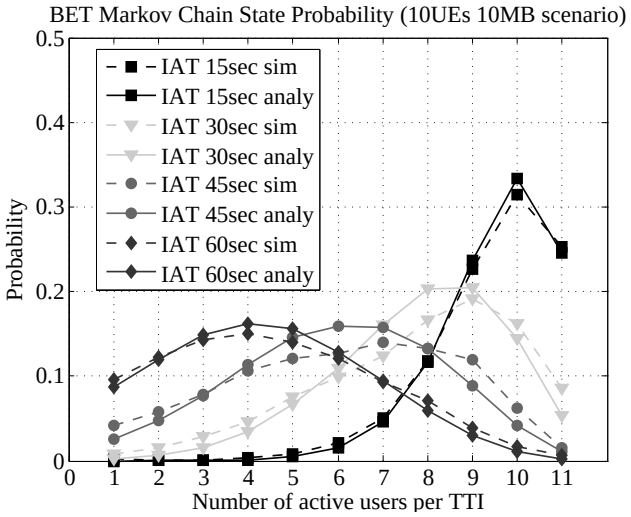


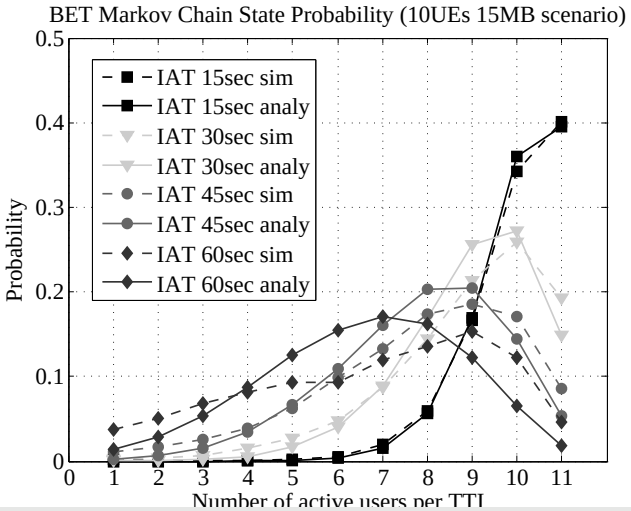




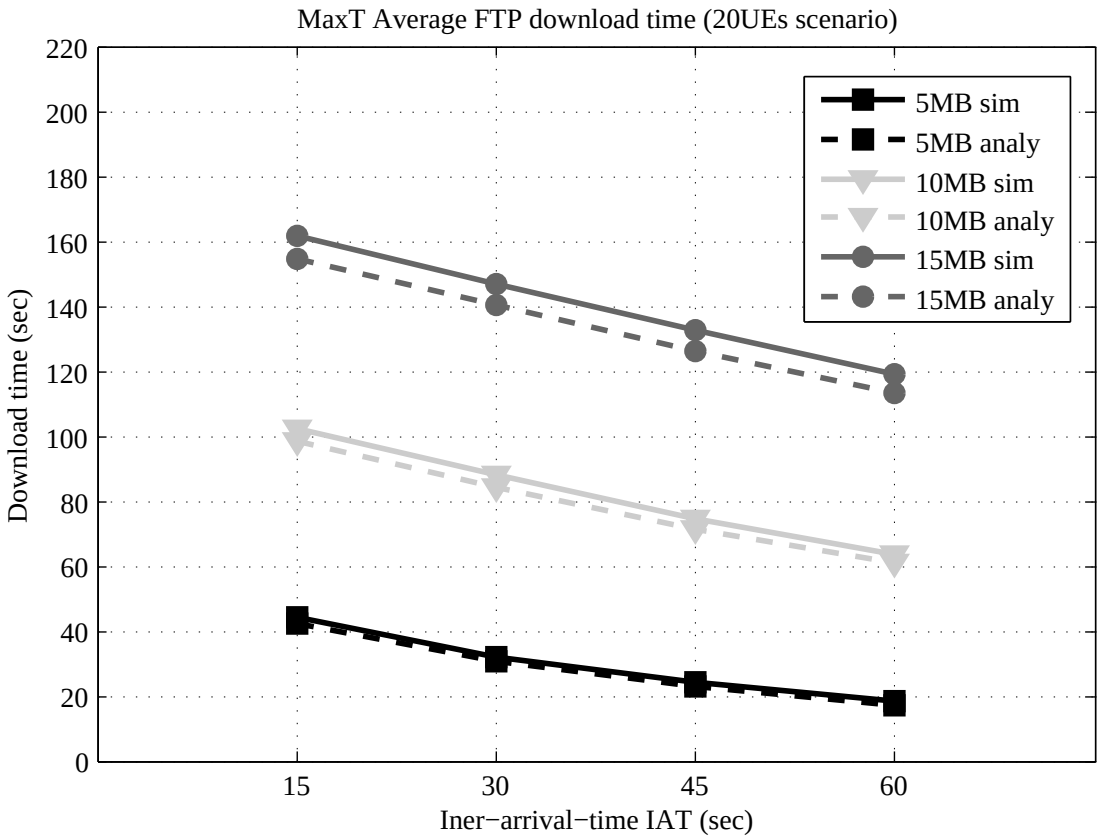
BET: State Probability



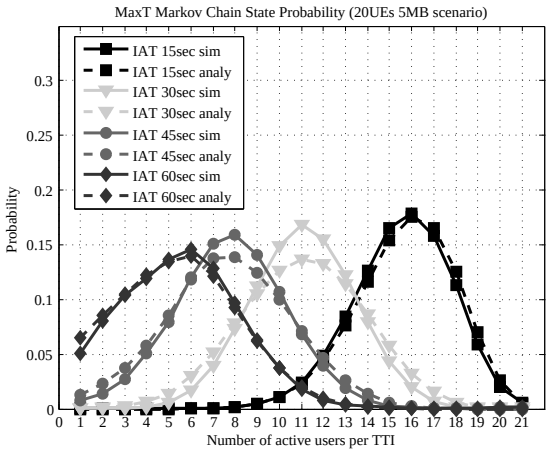


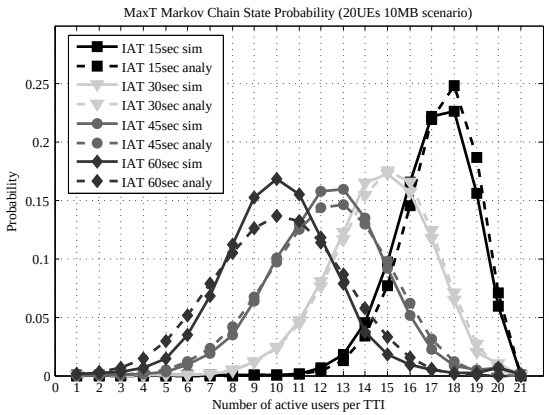


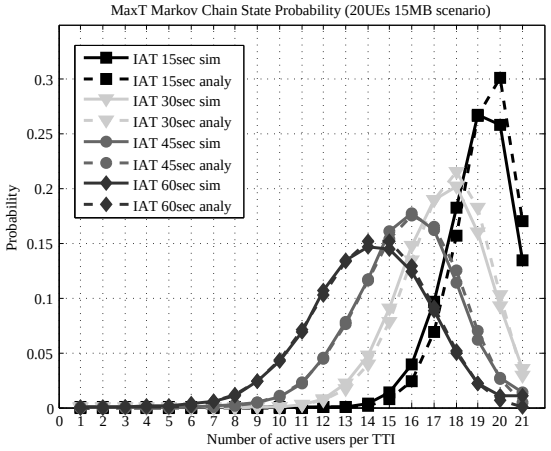
MaxT: FTP Download Time



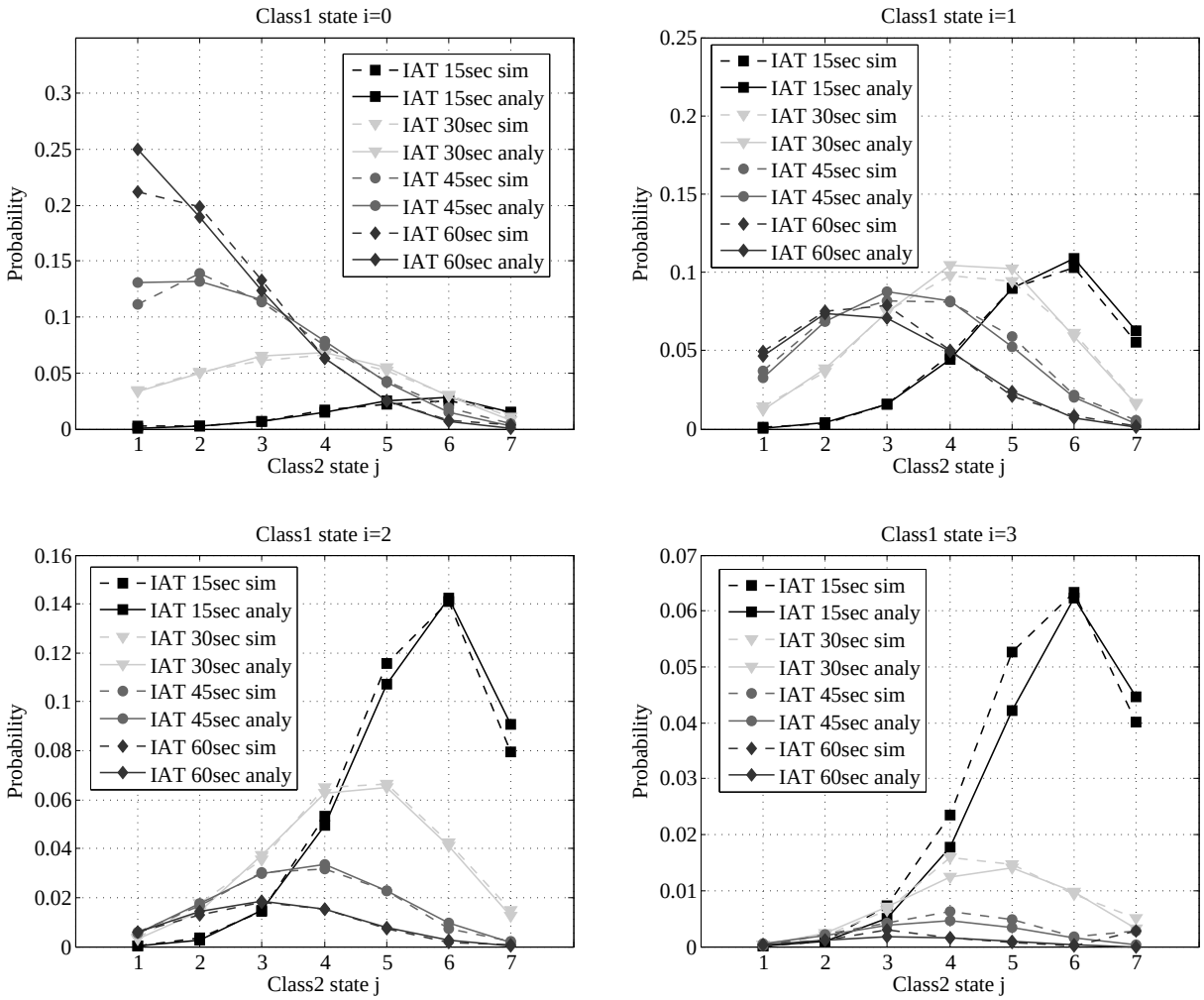
MaxT: State Probability







Analysis1: State Probability



Analysis2: State Probability